Statistics

Fall 2022

Lecture 16

Feb 19-8:47 AM

SG 14-17

Random variable x with prob. dist. $P(x)$

what is a prob. dist. $P(x)$?

It is a way to provide prob. of all possible outcomes.

1) $0 \le P(x) \le 1$

2) $\sum P(x) = 1$

3) $P(x) = 1 \Leftrightarrow$ Sure event

4) $P(x) = 0 \Leftrightarrow$ Impossible event

5) $0 < P(x) \le .05 \Leftrightarrow$ Rare event

$P(x)$ can be given / found

1) in the form of a table

2) in the form of a graph

3) in the form of a formula

4) by the concept of probability.

Recall that numerical data type were

1) Discrete (Countable)

2) Continuous (Measureable)

Nov 17-6:01 AM

Let x be a discrete random variable with Prob. dist. $P(x)$:

x	$P(x)$
1	.15
2	.45
3	.35
4	.05

1) Verify that $\sum P(x) = 1$.

$.15 + .45 + .35 + .05 = 1$ ✓

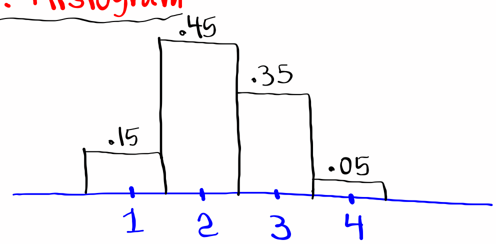
2) Find $P(x=2 \text{ or } x=3)$.

$= P(x=2) + P(x=3) = .45 + .35 = \boxed{.8}$

3) Draw Prob. dist. histogram

$x \to$ Midpoint

$P(x) \to$ Rel. F.



Nov 17-6:08 AM

Consider the chart below for random variable x with prob. dist $P(x)$:

x	$P(x)$
1	.05
2	.15
3	.35
4	.25
5	.20

1) Find $P(x=5)$

$= 1 - [.05 + .15 + .35 + .25]$

$= 1 - .8 = \boxed{.2}$

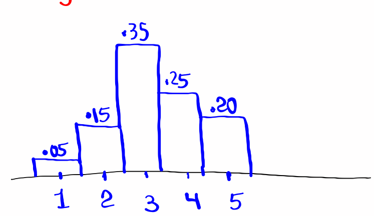
2) Find $P(x \geq 2)$

$= 1 - P(x=1) = 1 - .05 = \boxed{.95}$

3) Draw Prob. dist. histogram.

$x \to$ Midpoint

$P(x) \to$ Rel. F.



4) $P(x \leq 4)$

$= 1 - P(x=5) = 1 - .2 = \boxed{.8}$

Nov 17-6:14 AM

Random variable x with Prob. dist. $P(x)$:

Mean μ "mu"

Variance σ^2 "sigma^2"

Standard deviation σ "sigma"

$\mu = \sum x P(x)$

$\sigma^2 = \sum x^2 p(x) - \mu^2$

$\sigma = \sqrt{\sigma^2}$

Complete the chart below:

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.2	.2	.2
2	.5	1.0	2.0
3	.3	.9	2.7

$\sum p(x) = 1$

$\sum x p(x) = 2.1$

$\sum x^2 p(x) = 4.9$

$\mu = \sum x p(x) = \boxed{2.1}$

$\sigma^2 = \sum x^2 p(x) - \mu^2$

$= 4.9 - 2.1^2 = \boxed{.49}$

$\sigma = \sqrt{\sigma^2} = \sqrt{.49} = \boxed{.7}$

Nov 17-6:23 AM

How to find μ, σ, and σ^2 using TI:

$x \to L1$

$P(x) \to L2$

L1	L2
1	.2
2	.5
3	.3

$\boxed{\text{STAT}}\ \boxed{\to}$ CALC

$\boxed{\text{1: 1-Var Stats}}$

L1, L2 $\boxed{\text{enter}}$

$\boxed{7}$

List: L1

FreqList: L2

$\boxed{\text{Calculate}}$

$\mu = \bar{x} = 2.1$

$\sigma = \sigma_x = .7$

$n = 1$ ✓

For σ^2: $\sigma^2 = .49$

$\boxed{\text{VARS}}\ \boxed{\text{5: Statistics}}$

$\boxed{4: \sigma_x}\ \boxed{x^2}\ \boxed{\text{Enter}}$

for reduced fraction

$\boxed{\text{Math}}\ \boxed{\text{1: ▶Frac}}\ \boxed{\text{Enter}}$

$\sigma^2 = \frac{49}{100}$

Nov 17-6:33 AM

Consider the chart below for discrete random variable x with prob. dist. $P(x)$:

x	$P(x)$
1	.05
2	.25
3	.40
4	.15
5	.10
6	.05

1) Find $P(x=1)$

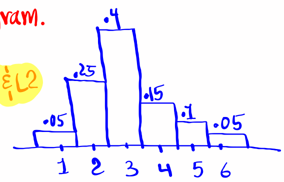
$=1-[.25+.40+.15+.10+.05]$

$=1-.95=\boxed{.05}$

2) Find $P(x\geq 2)$

$=1-P(x=1)=1-.05=\boxed{.95}$

3) Draw Prob. dist. histogram.



4) Use 1-Var Stats with L1&L2

$x\to L1$, $P(x)\to L2$ to find

$\mu=3.15$

$\sigma=1.195$

$n=1$

σ^2(Reduced Fraction) $=\frac{571}{400}$

95% Range

$\mu\approx 3$, $\sigma\approx 1$

68% Range: $\mu\pm\sigma$

$3\pm1 \Rightarrow \boxed{2 \text{ to } 4}$

Usual Range: $\mu\pm2\sigma$

$3\pm2(1)\Rightarrow \boxed{1 \text{ to } 5}$

Nov 17-6:40 AM

There are 2 females and 3 males.

Randomly select 2 people.

MM

MF

FM

FF

Let x be # of females selected

$MM \to x=0$

$\left.\begin{array}{l} MF \\ FM \end{array}\right\} \to x=1$

$FF \to x=2$

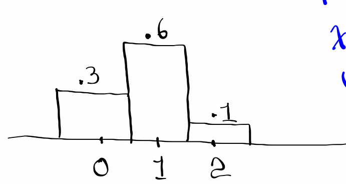
x	$P(x)$
0	.3
1	.6
2	.1

$P(x=0)=P(MM)=\frac{3}{5}\cdot\frac{2}{4}=\boxed{.3}$

$P(x=1)=P(MF \text{ or } FM)$

$=2\cdot\frac{3}{5}\cdot\frac{2}{4}=.6$

$P(x=2)=P(FF)=\frac{2}{5}\cdot\frac{1}{4}=\boxed{.1}$



$x\to L1$, $P(x)\to L2$

Use 1-Var Stats with L1&L2

$\mu=.8$ $\sigma=.6$ $n=$

σ^2(Reduced Fraction) $=\frac{9}{25}$

Nov 17-7:06 AM

A piggy bank has 4 nickels and 2 quarters.

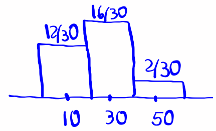
Randomly take 2 Coins. No replacement

NN → Total = 10 ¢

NQ
QN → Total = 30¢

QQ → Total = 50 ¢

Total ¢	P(Total ¢)
10	12/30
30	16/30
50	2/30



$P(T=10¢) = P(NN) = \frac{4}{6} \cdot \frac{3}{5} = \frac{12}{30}$

$P(T=50¢) = P(QQ) = \frac{2}{6} \cdot \frac{1}{5} = \frac{2}{30}$

$P(T=30¢) = P(NQ \text{ or } QN) = 2 \cdot \frac{4}{6} \cdot \frac{2}{5} = \frac{16}{30}$

Total → L1 , P(Total) → L1

Use 1-Var Stats with L1 & L2

$\mu = \bar{x} \approx 23$

$\sigma = \sigma_x \approx 12$

$n = 1$

σ^2 in reduced fraction

$\boxed{\sigma^2 = \frac{1280}{9}}$

68% Range ⇒ $\mu \pm \sigma = 23 \pm 12$

⇒ $\boxed{11 \text{ to } 35}$

95% Range ⇒ $\mu \pm 2\sigma = 23 \pm 2(12)$ ⇒ $\boxed{-1 \text{ to } 47}$

Usual Range 23 ± 24

Nov 17-7:17 AM

Application:

Expected Value ⇒ μ ⇒ $\bar{x}$

Consider a class of 20 students.

20 Tkts were Sold for \$10 each.

One ticket is drawn, the winner gets a calc. worth \$120. What is expected Value/Tkt for the fundraiser?

	Net	P(Net)
Winner	10 − 120	1/20
losers	10 − 0	19/20

Net → L1

P(Net) → L2

1-Var Stats with L1 & L2

Expected Value = $\mu = \bar{x}$

\$ 4

Fundraisers make \$4 Per ticket.

Nov 17-7:31 AM

A deck of cards has 40 cards, 6 face cards, and 2 aces.

Pay me \$10 and draw a card,

if you draw an ace ⇒ I give you \$50

If you draw a face ⇒ I give you \$20

Any other card, ⇒ I give you nothing.

Find expected value per bet for the house.

Net	P(Net)	
10 − 50	2/40	Ace
10 − 20	6/40	Face
10 − 0	32/40	Any other Card

Net → L1

P(Net) → L2

1-Var Stats with L1 & L2

$E.V. = \mu = \bar{x} =$ \$4.5

Nov 17-7:38 AM

Airline sells insurance for the luggage at \$100.

Any damages ⇒ Airline pays \$1000

Prob. of damage to any luggage is .2%.

Find expected value per policy sold for the airline.

Net	P(Net)	
100 − 1000	.2% = .002	Damage
100 − 0	99.8% = .998	$\overline{\text{Damage}}$

.2% = .002

1 − .002 = .998

Net → L1

P(Net) → L2

1-Var Stats with L1 & L2

$E.V. = \mu = \bar{x}$

\$98

Airline makes \$98 per policy sold

Nov 17-7:47 AM

Use Your Calc to find

1) ${}_{10}C_3 \cdot (.6)^3 \cdot (.4)^7 \approx \boxed{.042}$

2) ${}_{20}C_4 \cdot (.5)^4 \cdot (.5)^{16} \approx \boxed{.005}$

Suppose $P(A) = .8$ $P(B) = .5$ $P(A \text{ and } B) = .45$

1) $P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)} = \dfrac{.45}{.8} = .5625 \approx .563$

2) $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{.45}{.5} = \boxed{.9}$

Nov 17-8:06 AM

4 Red , 6 Blue Balls

Randomly Select 3 balls, no replacement

$P(\text{All Red}) = \dfrac{{}_4C_3}{{}_{10}C_3} = \boxed{\dfrac{1}{30}}$

$P(\text{All Blue}) = \dfrac{{}_6C_3}{{}_{10}C_3} = \boxed{\dfrac{1}{6}}$

$P(\text{at least 1 red ball}) = 1 - P(\text{No red})$
$= 1 - P(\text{All Blue}) = 1 - \dfrac{1}{6} = \boxed{\dfrac{5}{6}}$

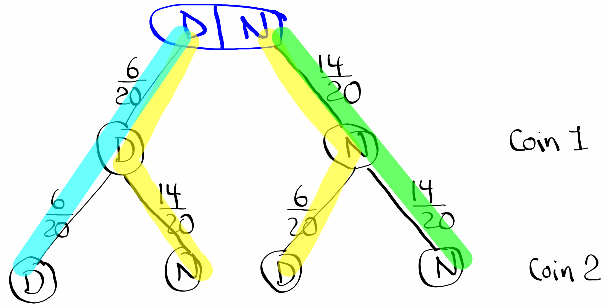
$P(\text{at least 1 blue ball}) = 1 - P(\text{No blue})$
$= 1 - P(\text{All red})$
$= 1 - \dfrac{1}{30} = \boxed{\dfrac{29}{30}}$

Nov 17-8:13 AM

6 Dimes, 14 Nickels,

Select 2 Coins with replacement.

Construct Tree diagram



$P(1D \,\&\, 1N) = 2 \cdot \frac{6}{20} \cdot \frac{14}{20} = \boxed{\frac{21}{50}} = \boxed{.42}$ ✓

$P(2 \text{ Dimes}) = \frac{6}{20} \cdot \frac{6}{20} = \boxed{\frac{9}{100}} = \boxed{.09}$ ✓

$P(2 \text{ Nickels}) = \frac{14}{20} \cdot \frac{14}{20} = \boxed{\frac{49}{100}} = \boxed{.49}$ ✓

Nov 17-8:19 AM